# IMU

BULLETIN

OF THE

INTERNATIONAL MATHEMATICAL UNION

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Secretariat

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SWEDEN

## INTERNATIONAL MATHEMATICAL UNION

Executive Committee

(1 January 1971 - 31 December 1974)

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| President: | Professor K. Chandrasekharan |
| Vice-Presidents: | Professor Nathan Jacobson |
| | Academician L.S. Pontrjagin |
| Secretary: | Professor Otto Frostman |
| Members: | Professor M.F. Atiyah |
| | Professor Y. Kawada |
| | Professor N.H. Kuiper |
| | Academician M. Nicolescu |
| | Professor E. Vesentini |
| Past President: | Professor H. Cartan |

## Report of the 31st Meeting of the Executive Committee

## The Royal Society, London, June 8-9, 1972

The following members of the E.C. were present at the meeting:
Professor K. Chandrasekharan (President), Professor H. Cartan (Past President), Academician L.S. Pontrjagin (Vice-President), Professor O. Frostman (Secretary), Professor M.F. Atiyah, Professor Y. Kawada, Professor N.H. Kuiper, Academician M. Nicolescu and Professor E. Vesentini (Members).

The President made a reference to the death, on June 6, 1972, of Professor A. Adrian Albert, one of the Vice-Presidents of IMU, and asked the members of the E.C. to honour his memory by observing a minute's silence. This was done.

### Approval of Reports.

The financial report for 1971, and the report to the National Adhering Organizations for 1971, the proofs of which were given to the members, were approved.

### IMU Lectures.

In February-March 1972, IMU Lectures were given by Professor F. Hirzebruch at the University of Tokyo. Professor Pontrjagin informed the E.C. that the IMU Lectures by Professor J.-L. Lions would take place in Moscow in the Fall of 1972.

### Mathematical-Physical Conference in Moscow.

The conference on Mathematical Problems of Quantum Field Theory and Quantum Statistics will be held in Moscow on December 12-19, 1972. The grant of $ 1.000, previously proposed, was confirmed.

### International Symposium in Israel.

The Secretary informed the E.C. that the symposium on the Geometry of Linear Spaces and Partial Differential Equations was due to be held on June 19-28, 1972.

### Symposium in Bombay.

The symposium on Discrete subgroups of Lie groups and applications to moduli, planned for January 1972, was postponed until January 1973.

**Second International Congress on Mathematical Education.**

This congress, to be held at Exeter, August 29 - September 2, 1972, had been discussed already at the Zürich meeting, but no financial commitments had been made.

In view of ICMI's own funds, and of contributions received from other sources, the E.C. voted a grant of $ 1.500 for the Congress.

**Mathematical Congress in Bulgaria.**

The Secretary informed the E.C. about the invitation to participate in this congress.

**Conference in Cambridge.**

The E.C. decided to sponsor the international geometry conference (with Prof. J.W.S. Cassels as chairman and Prof. J.F. Adams as secretary of the organizing committee) to be held in Cambridge, July 3-10, 1973, and to give financial support of up to $ 2.000 for the conference. The E.C. further invited Professors H. Cartan and G. de Rham, the two former Presidents of the Union, to attend the conference on behalf of the Union.

**International Colloquium in Hungary.**

The E.C. decided to sponsor the international colloquium on Infinite and Finite Sets, to be held in Hungary in June 1973. It was further decided to give financial support to this colloquium of up to $ 2.000 for travel grants. The E.C. nominated Professors R. Rado and E. Specker as the IMU representatives on the organizing committee.

**Conference in Poland.**

Professor Pontrjagin informed the E.C. about the projected conference in Poland at the beginning of 1974 on Optimization and Control Problems. Normally the Union would not sponsor anything during a Congress year, but as this conference would be interdisciplinary, the Union might approach ICSU for financial aid, if and when an application was received.

**The World Directory of Mathematicians.**

The Secretary informed the E.C. about the total costs ($ 4-5.000) for the printing, proof-reading and free copies (about 50) of the 4th edition of the WDM. He had tried to get an offer for the next edition from the same printers as last time

(Almqvist & Wiksell, Uppsala, Sweden), but so far the discussions had only been preliminary. However, as the printers had all the material for the 4th edition, the costs ought to decrease. The E.C. decided

a) to let Almqvist & Wiksell publish the 5th edition of the WDM,

b) that the edition be issued well in advance of the next ICM, and not later than Spring 1974,

c) to authorize the Secretary to pursue the negotiations with Almqvist & Wiksell on the financial questions, the final decisions to be taken later on, and

d) to authorize the Secretary to ask the printers for the latest date of receiving the material, so that he could write to the NAO asking them to begin preparations in proper time (with copies to the members of the E.C.).

**The IMU Bulletin.**

It was agreed to publish two numbers of the IMU Bulletin every year. It was understood that the cost of publication would be slightly more than the $ 300 a year allocated by the General Assembly for IMU News.

**The Canberra circulars.**

The E.C. expressed its appreciation of the circulars brought out by Professor B.H. Neumann and sent to different centres of mathematics and hoped that the initiative taken would be carried on.

**New structure of ICSU.**

The President gave a summary of the proposed changes in the structure of ICSU. The President, as the representative of IMU in ICSU, would exercise the Union's vote. It was decided that, if the money was available, IMU could send two more delegates, one of them being Professor H. Cartan (Past President).

**UNESCO proposals.**

The E.C. was informed of the discussions that took place at a conference recently organized by Unesco in Paris. One of the results of the conference was the appointment by Unesco of an Evaluation Team to visit the Trieste Centre and make a report. The E.C. wished the team success in its task, and offered to be of help in considering their report at a later stage.

**Fields Medals Committee.**

A committee was appointed for the award of the Fields Medals at the Vancouver Congress.

**Site Committee for the International Congress in 1978.**

A committe was appointed for the selection of the site of the 1978 Congress.

**Application of membership from South Korea.**

The E.C. unanimously decided to recommend the admission of South Korea as a member of the Union in group I. Furthermore, if by mutual agreement of both North Korea and South Korea, these countries would like to change their names to Democratic People's Republic of Korea and Republic of Korea respectively, the E.C. considered that the best time to do so would be at the General Assembly of IMU.

**Change of groups of adherence of Bulgaria and Sweden.**

It was reported that the change of the group of adherence of Bulgaria from I to II, and of Sweden from II to III, had been approved by the members of the Union by postal ballot. The changes would be effective from January 1, 1973.

**Change of group of adherence of Australia.**

The E.C. was in favour of Australia changing its group of adherence from group II to group III. The Secretary was authorized to conduct a postal ballot by the members of the Union if and when an official application for the change was received.

**Election of a new Vice-President.**

It was unanimously decided that Professor Nathan Jacobson should be asked to fill the vacancy in the office of Vice-President caused by the death of Professor Albert. If Professor Jacobson accepted the nomination, the members of the Union will vote on the nomination by postal ballot. (This has since been done, and Professor Jacobson has been elected, December 1972).

**Informal**

There was an informal session of the E.C. at which the proposed new structure of ICSU was discussed. The Executive Secretary of ICSU, Mr. F.W.G. Baker, attended the session by invitation and explained the various proposals for the revision of ICSU Statutes.

## Abraham Adrian ALBERT (1905 - 1972)

**by N. Jacobson**

Professor Adrian Albert, one of the two Vice Presidents of IMU, died on June 6, 1972 almost exactly one year after his retirement as Dean of Physical Sciences at the University of Chicago. He was in poor health for almost a year and, though he attended a number of conferences and meetings of the IMU during this period, it became increasingly apparent that he was not well. Considering his vigor during the period immediately preceding his retirement from the deanship, the rapid deterioration of his health and subsequent death were a deep shock to his many friends.

Albert belongs to the second generation of American mathematicians following the first generation of E.H. Moore, Dickson, Wedderburn (Scottish born), Veblen, etc. His formation was wholly American. He was a student of Dickson's and a follower of Wedderburn's, particularly in his early important work. In this brief account of his work we shall confine our attention to three areas: I. Associative algebras, II. Riemann matrices. III. Jordan algebras. These include nearly all of his best work. A fuller account of his work will appear in the Bulletin, A.M.S.

**I. Associative algebras.** The Wedderburn structure theorems of 1908 focused attention on the study of division algebras. Much of Albert's early work was concerned with the problem of classifying finite dimensional division algebras, with its main objective the determination and classification of these algebras over a number field. Parallel to his work, the German algebraists of the same period (the end of the twenties and beginning of the thirties) were attacking the same problems. As a result there was a substantial overlap between Albert's results and those obtained by R. Brauer, by Hasse and by Emmy Noether.

It was recognized early that the problem of classifying finite dimensional division algebras over number fields had two quite separate aspects: a purely algebraic one concerned with properties of algebras valid for all base fields and an arithmetic one exploiting the arithmetic of number fields. Albert's main contributions were in the first of these directions. He obtained independently all the results on splitting fields, centralizers of simple subalgebras of central simple algebras, extensions of isomorphisms of such subalgebras, tensor products of central simple algebras which were needed to solve the problem. In one of his best papers (1) in this field, he essentially rediscovered the Brauer group about a year or two after Brauer. In this paper he proved Brauer's theorem that if $n$ is the degree of a central division algebra $A$ ($n^2$ the dimension) then the nth direct power of $A$ is a complete matrix algebra. Albert's proof is based on Wedderburn's norm criterion for cyclic algebras and is different from the now standard one using crossed products or Brauer's based on his type of factor sets.

The crowning achievements of the development of the theory of algebras of the nineteen thirties were the proof that every finite dimensional central division algebra over a number field is cyclic (that is, has a maximal cyclic subfield and hence has a simple form which was first defined by Dickson), and the determination of the structure of the Brauer group of algebras over a number field. Besides the algebraic results to which we have alluded, these required the theory of central simple algebras over p-adic fields which was due to Hasse, Hasse's norm theorem ("the Hasse principle"), and the Grünwald existence theorem for certain types of cyclic extensions of a number field. (Though is was discovered almost thirty years later by S. Wang that Grünwald's formulation was incorrect, his error did not affect the theorem on algebras.) The first proof of the cyclic structure theorem for division algebras over a number field was given by Brauer, Hasse and Noether (20). However, it seemed appropriate that Albert should share in the honor of the achievement, and at Hasse's suggestion, a joint paper (17) was published by Albert and Hasse giving another proof and the historical background of the problem.

The first example of a non-cyclic division algebra is due to Brauer. This was a tensor product of two quaternion algebras. Albert then gave a construction of a division algebra of degree four which was neither cyclic nor a tensor product of cyclic algebras (2). Also he proved the beautiful theorem that all division algebras of degree four are crossed products, the next step after Wedderburn's theorem that the degree three division algebras are cyclic (3). Albert in (4) and, independently, Teichmüller in (22) determined the structure of p-algebras, defined to be central simple algebras of characteristic p and exponent a power of p.

**II. Riemann matrices.** Albert's work on division algebras was strongly motivated by the problem of Riemann matrices which had its origin in algebraic geometry. A Riemann matrix $\omega$ is a period matrix for the abelian integrals of the first kind on the Riemann surface of an algebraic curve. If the genus is p, $\omega$ is a p x 2p complex matrix for which there exists a 2p x 2p rational matrix C such that $\omega\, C\, \omega' = 0$ and $i\omega\, C\, \omega'$ is positive definite hermitian. A 2p x 2p rational matrix A is called a multiplication of $\omega$ if there exists a p x p complex matrix $\alpha$ such that $\alpha\omega = \omega A$. These arise from algebraic correspondences on the Riemann surface which defines $\omega$. The set of multiplications is an algebra over the rationals and one has the basic problem of determining the structure of these algebras and the converse problem of constructing Riemann matrices from algebras satisfying the necessary conditions for multiplication algebras. The early work on these problems is due to Poincaré, Klein, Scorza, Lefschetz and Rosati. These reduced the problems to so-called pure Riemann matrices whose multiplication algebras were division algebras with involutions (anti-automorphisms of period one or two).

Albert began his study of Riemann matrices and their multiplication algebras in 1930. He unfolded the theory in a series of papers nearly all of which were based on some progress in the theory of algebras. These culminated in complete solutions of the problems in (5), (6), and (7). In 1934 Weyl gave a new definition of Riemann matrices which led naturally to a generalization (23). Albert generalized this still further and gave a complete solution of the problems for these generalized Riemann matrices in (8). Subsequently Weyl in (24) gave an alternative approach to these questions based on Brauer's method of factor sets.

In the course of his study of Riemann matrices, Albert was led to the study of simple algebras with an involution. His results on these have not been superseded and his account of the theory of involutorial simple algebras which appears in (9) remains the standard reference on this subject.

**III. Jordan algebras.** After about 1940 most of Albert's research concerned non-associative algebras. He obtained some striking results on alternative algebras, introduced a general notion of isotopy and, together with M. Frank, discovered some new classes of simple Lie algebras of characteristic p. Perhaps his most important work in this field was on the structure theory of Jordan algebras.

In 1932, the physicist P. Jordan initiated the study of associative algebras relative to the "Jordan product" $a \cdot b = \frac{1}{2}(ab + ba)$, ab the given associative product. He noted that the product $a \cdot b$ satisfied the identities: $a \cdot b = b \cdot a$ and $(a^2 \cdot b) \cdot a = a^2 \cdot (b \cdot a)$. This led to the definition of a new class of algebras, called Jordan algebras by Albert, the algebras satisfying these two identities. The structure theory for these algebras was launched in 1934 in a joint paper by Jordan, von Neumann and Wigner (21). In this they determined the structure of finite dimensional Jordan algebras over the reals which were formally real in the sense of Artin-Schreier. They showed that these algebras were direct sums of simple ones and they determined the simple ones. All of the simple ones except one were special in the sense that they were subalgebras of algebras obtained from associative ones by replacing the given multiplication by $a \cdot b$. The one possible exception, which they denoted as $M_3 8$, was the vector space of 3 x 3 hermitian Cayley (octonion) matrices with the product $A \cdot B = \frac{1}{2}(AB + BA)$, AB the usual matrix product. In 1934, in (10), Albert showed that $M_3{}^8$ was indeed exceptional ( = not special). In 1959, in (19), Albert and Paige proved the surprising result that $M_3{}^8$ is not a homomorphic image of a special Jordan algebra.

In three papers, (11), (12), (13), appearing from 1946-1950 Albert developed a structure theory of finite dimensional Jordan algebras over an arbritrary field of characteristic $\neq 2$. He defined the radical as the maximal nil ideal and obtained its basic properties. He proved that algebras with 0 radical are direct sums of

simple ones and he classified the simple ones over an algebraically closed field. In the third paper he obtained the very surprising result that any commutative simple ring of characteristics $\neq 2, 3, 5$ which is power associative in the sense that subrings generated by single elements are associative is Jordan. This result is valid without any finiteness restrictions. In (14) and (15) he gave a determination of exceptional Jordan division algebras, and in (18), Albert and Jacobson derived isomorphism conditions for exceptional simple algebras which are not division algebras. This gave a classification of exceptional simple Jordan algebras over number fields and as a consequence, also a classification of exceptional Lie algebras of type $F_4$ over a number field.

It is interesting that at the end of his career Albert returned to a problem on associative algebras on which he had worked almost forty years earlier: Are central division algebras of prime degree $> 3$ cyclic? His retiring presidential address of the A.M.S. was devoted to this problem (16).

Perhaps the most striking of Albert's qualities as a mathematician were his power, expecially his fantastic computational skill, and his tenacity in pursuing difficult problems of recognized interest. While he regarded himself as a "pure" algebraist, some of his best work arose from applications to other fields. As anyone who knew him will attest, Albert had wonderful personal qualities: directness, complete honesty without pretense, decisiveness and generosity. These made him very effective in his administrative positions and in his work in IMU.

**References**

Albert: (1) Trans. AMS v. 33, 620. (2) ibid, v. 35, 112. (3) ibid, v. 31, 253. (4) ibid, v. 40, 112. (5) Annals of Math. v. 35, 1. (6) ibid, 500. (7) ibid, v. 36, 376. (8) ibid, v. 36, 886. (9) **Structure of Algebras,** AMS Colloq. Publ. XXIV, 1939. (10) Annals of Math. v. 35, 65. (11) Trans. AMS v. 59, 524. (12) Annals of Math. v. 48, 546. (13) Trans. AMS v. 69, 503. (14) Annals of Math. v. 67, 1. (15) Pacific J. Math. v. 15, 799. (16) Proc. AMS v. 16, 799.

Albert and Hasse: (17) Trans. AMS v. 34, 722.

Albert and Jacobson: (18) Annals of Math. v. 66, 400.

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Brauer, Hasse and Noether: (20) J. für Math. v. 167, 399.

Jordan, von Neumann and Wigner: (21) Annals of Math. v. 36, 29.

Teichmüller: (22) Deutsche Math. (1936), 92.

Weyl: (23) Annals of Math. v. 35, 714. (24) ibid, v. 37, 709.

## 14th General Assembly of ICSU

The President, the Past President, and the Secretary of IMU represented the Union at the 14th General Assembly of ICSU.

The Assembly met at Helsinki on 16-20 September 1972 and was preceded by an Extra-ordinary session devoted to the task of revising the constitution of ICSU. The new constitution is now in force. It will be printed in the ICSU Year Book.

There is now an Executive Board, a General Committee, and a General Assembly. The Executive Board of nine corresponds, with some changes, to the old committee of 8 Officers. It consists of the President, Vice-President, Secretary-General, Treasurer, Past-President, together with two representatives of the Unions and two representatives of the National Members. (These four were formerly designated Vice-Presidents, but not any longer). The General Committee corresponds to the old Executive Committee and is supposed to be a policy-making body. Special Committees and Commissions of ICSU could send observers to the meetings of the General Committee at their own cost.

The International Union of Pharmacology was admitted as a new Union Member. ICSU's policy on the Free Circulation of scientists was reiterated. COSTED (Committee on Science and Technology in Developing Countries) was reconstituted with specific terms of reference, and with a special provision for "correspondents" from all ICSU bodies.

The new Executive Board consists of Prof. J. Coulomb (President), Prof. H. Brown (Vice-President), Prof. F.A. Stafleu (Secretary General), Prof. N.B. Cacciapuoti (Treasurer), Prof. V.A. Ambartsumian (Past-President), Prof. C. de Jager, Prof. F.G. Young, Prof. J. Sahade and Prof. F.B. Straub as members.

## ICMI General Assembly

A meeting of the General Assembly of ICMI was held at Exeter, during the Second International Congress on Mathematical Education, on Friday, 1 September 1972. About 60 members were present, and the President, Professor Sir James Lighthill, was in the chair. The Assembly elected Dr. E.A. Maxwell (U.K.) as Secretary, and Professors S. Iyanaga (Japan) and J. Suranyi (Hungary) as Vice-Presidents. As additional members of the Executive Committee were elected: Professors H. Freudenthal (The Netherlands), Dr. H. Pollak (U.S.A.) and Academician S.L. Sobolev (U.S.S.R.).

ICMI sponsorship was given to a number of international symposia due to be held in the next three years:in Luxembourg in May-June 1973, on "New Topics in applicable mathematics in secondary schools", In Hungary (at Eger) on 18-22 June 1973 on "Theoretical problems of Teaching Mathematics in the Primary Schools", in Poland in 1974 on "Teaching children of age 5-11", in Scandinavia in 1974 or 75 on "Aspects of Geometry teaching at School Level". Plans for holding "regional symposia" (i) in Africa, on "Mathematics and Language", (ii) in the Japan-Siberia-Australia-New Zealand region, and (iii) in India on "Integrated curriculum development including applications of mathematics relevant to problems of developing countries", were also discussed. It was proposed that steps should be taken for a joint ICMI-IFIPS (International Federation of Information Processing Societies) symposium on "Computer Science in Schools". A minimum scale of ICMI financial support to every supported symposium was agreed at $ 500.

Further details can be had from the Secretary of ICMI. A report of the Assembly is expected to appear in the L'Enseignement Mathématique, an official organ of ICMI.

### Exeter Congress on Mathematical Education

The congress was held on August 29 - September 2, 1972, and was attended by 1400 members from about 70 countries, and was presided over by Professor Lighthill. The primary work of the congress was centred into the 38 working groups on different specialized aspects of mathematical education, centred either on the treatment of particular parts of mathematics at particular levels of education or on particular aspects of method or technology and on the fundamental studies underlying the choice of method. The proceedings of the congress will be published, and given free of charge to the congress members. They will also be on sale to the general public.

## ANNOUNCEMENTS

### IMU Lectures 1973

Applications for IMU Lectures in 1973 should reach the Secretary of IMU before May 15, 1973.

### International Congress of Mathematicians 1978

Invitations from prospective hosts for the ICM 1978 should reach the Secretary of IMU before December 1, 1973.

### Salem prize

The Salem Prize for 1972 was awarded to Dr. Thomas Körner, of the University of Cambridge (England), for his work on perfect sets and trigonometric series. The prize, established in 1968, is given every year to a young mathematician who is judged to have done outstanding work on Fourier series and related topics. Previous recipients were : Dr. Nicholas Varopoulos in 1968. Dr. Richard Hunt in 1969, Dr. Yves Meyer in 1970 and Dr. Charles Fefferman in 1971. The jury consisted of Professor A. Zygmund, Professor C. Pisot and Professor J.-P. Kahane.